JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 7, No. 1, Jan.–March 1993

Drag and Heat Transfer Correlations for Rarefied Hypersonic Flow Past Flat Plates

Michael S. Woronowicz*
Vigyan Research Associates, Hampton, Virginia 23666
and
Donald Baganoff†
Stanford University, Stanford, California 94305

The independent and dependent variables associated with drag and heat transfer to a flat plate at zero incidence, in high-speed, rarefied flow, are analyzed anew to reflect the importance of kinetic effects occurring near the plate surface on energy and momentum transfer, rather than following arguments normally used to describe continuum, higher density flowfields. A new parameter, the wall Knudsen number $Kn_{x,w}$, based on an estimate of the mean free-path length of molecules having just interacted with the surface of the plate, is introduced and used to correlate published drag and heat transfer data. The new parameter is shown to provide better correlation than either the viscous interaction parameter $\bar{\chi}_\infty$, or the widely-used slip parameter $\bar{V}_\infty$, for drag and heat transfer data over a wide range of Mach numbers, Reynolds numbers, and plate-to-freestream stagnation temperature ratios.

Nomenclature

A = area of plate
a = speed of sound
C = Chapman-Rubesin constant
C_D = drag coefficient
C_H = Stanton number
C_Q = heat transfer coefficient
Kn = Knudsen number
$\overline{Kn}_{x,w}$ = reduced wall Knudsen number
M = Mach number
Pr = Prandtl number
p = pressure
R = gas constant
Re = Reynolds number
T = temperature
U = streamwise velocity
$\bar{V}_\infty$ = slip parameter
α = relates p_w to $\bar{\chi}_\infty$
γ = ratio of specific heats
λ = mean free-path length
μ = coefficient of viscosity
ξ = relates C_Q to C_H
ρ = density
$\bar{\chi}_\infty$ = viscous interaction parameter
ω = exponent in $\mu \propto T^\omega$

Subscripts
e = edge of boundary layer
fm = free-molecule conditions
ref = reference state
w = wall condition
x = distance from leading edge
0 = freestream stagnation conditions
∞ = freestream conditions

Superscript
* = denotes modified parameter

Introduction

In reviewing data presented in the literature concerning drag and heat transfer acting on flat plates in various flows, one is struck by the variety of parameters used to correlate results for particular experiments.[1–11] In incompressible flow, the drag of, and the heat transfer to, a flat plate at zero incidence, are generally expressible as functions of the Reynolds number $Re_{x,\infty}$, the Prandtl number Pr_∞, and the plate temperature T_w, because the physical problem is fairly straightforward.[12] However, for compressible flow, as the Reynolds number is decreased while holding Mach number fixed, the displacement effects of the boundary layer become important, and it becomes increasingly difficult to find suitable correlating parameters for such conditions.

A parameter frequently used to characterize flowfields with appreciable boundary-layer displacement is the viscous interaction parameter $\bar{\chi}_\infty$, which originates from an analysis of the pressure distribution on a flat plate.[13] Although it correlates pressure distributions fairly well,[13] it has not been as successful in correlating drag and heat transfer measurements. The viscous interaction parameter is defined by the relation

$$\bar{\chi}_\infty \equiv \frac{M_\infty^3\sqrt{C}}{\sqrt{Re_{x,\infty}}} \tag{1}$$

where C is the Chapman-Rubesin constant, given by

$$C \equiv \frac{\mu_\infty T_{\text{ref}}}{\mu_{\text{ref}} T_\infty} \tag{2}$$

and where "ref" denotes some reference state.

Another parameter, one that has met with some success in correlating drag and heat transfer measurements, is the so-called slip parameter $\bar{V}_\infty$. This parameter has been related to a ratio of length scales,[2,14] as well as a pressure coefficient expressed in terms of $\bar{\chi}_\infty$.[15] It is usually defined as

$$\bar{V}_\infty \equiv \frac{M_\infty\sqrt{C}}{\sqrt{Re_{x,\infty}}} = \frac{\bar{\chi}_\infty}{M_\infty^2} \tag{3}$$

Received Feb. 21, 1991, presented as Paper 91-1314 at the AIAA 26th Thermophysics Conference, Honolulu, HI, June 24–26, 1991; revision received Oct. 24, 1991; accepted for publication Nov. 25, 1991.

*Research Associate. Member AIAA.
†Professor, Department of Aeronautics and Astronautics. Member AIAA.

In the strong interaction regime, the connection between $\bar{V}_\infty$ and C_D can be shown as follows. The analytical result for drag in this regime is often represented by[13]

$$C_D \sqrt{(Re_{x,\infty}/C)} \propto \sqrt{\bar{\chi}_\infty} \tag{4}$$

Rearranging and using Eq. (3), we obtain

$$C_D \propto \frac{M_\infty^{3/2} C^{3/4}}{Re_{x,\infty}^{3/4}} = \left(\frac{M_\infty \sqrt{C}}{\sqrt{Re_{x,\infty}}}\right)^{3/2} = \bar{V}_\infty^{3/2} \tag{5}$$

We propose that one employ a length scale ratio different from $\bar{V}_\infty$; one formed by the ratio of the mean free-path length of molecules emitted from the plate surface to the running length of the plate from the leading edge. This definition (to be developed below), introduces a Knudsen number defined by the relation

$$Kn_{x,w} = \sqrt{\frac{\pi\gamma}{2}} \frac{M_\infty}{Re_{x,\infty}} \left(\frac{T_w}{T_\infty}\right)^{1/2+\omega} \tag{6}$$

In Eq. (6), γ is the ratio of specific heats, T_w is the temperature of the plate's surface, and ω is the exponent in a power law representation of viscosity, given by

$$(\mu/\mu_\infty) = [(T/T_\infty)^\omega] \tag{7}$$

We will demonstrate below that for a wide range of M_∞ and $Re_{x,\infty}$, the parameter defined by Eq. (6) correlates drag and heat transfer measurements better than Eq. (3) in the weak and strong viscous interaction regimes.

Wall Knudsen Number

Within one local mean free-path length of a body immersed in a gas there exists what is called a "Knudsen layer."[16] Within this layer, molecules are more likely to collide with the surface of the body than with each other. From the kinetic viewpoint, only collisions between gas molecules and the surface provide the physical mechanisms for drag and heat transfer. Under the rarefied conditions encountered in the viscous interaction regimes, the large local mean free-path lengths lead to rather thick Knudsen layers. We then anticipate that the mean free-path length of the newly-emitted molecules λ_w could be important in the basic physics determining flat plate drag and heat transfer. On this basis, we choose to introduce the Knudsen number defined by the nondimensional length scale

$$Kn_{x,w} \equiv (\lambda_w/x) \tag{8}$$

and employ it in correlating data associated with rarefied flows. In Eq. (8), x is the running length along the surface from the leading edge of the plate.

One may evaluate the wall Knudsen number $Kn_{x,w}$ using the following approach. From the kinetic theory expression relating viscosity and mean free-path length, where ρ and a are the local density and speed of sound, respectively, we have

$$\lambda = \sqrt{(\pi\gamma/2)}(\mu/\rho a) \tag{9}$$

On using definition (8) for an arbitrary point in the flow, we obtain

$$Kn_x = \sqrt{(\pi\gamma/2)}(M/Re_x) \tag{10}$$

The ratio of the mean free-path length at the surface to that in the freestream is obtained from Eq. (9), and it reads

$$\frac{\lambda_w}{\lambda_\infty} = \left(\frac{\mu_w}{\mu_\infty}\right)\left(\frac{\rho_\infty}{\rho_w}\right)\left(\frac{a_\infty}{a_w}\right) \tag{11}$$

Assuming the power law viscosity-temperature relationship (Eq. (7)), and on using $p = \rho RT$, we then have

$$\frac{\lambda_w}{\lambda_\infty} = \frac{Kn_{x,w}}{Kn_{x,\infty}} = \left(\frac{p_\infty}{p_w}\right)\left(\frac{T_w}{T_\infty}\right)^{1/2+\omega} \tag{12}$$

Finally, on using Eq. (10) evaluated at freestream conditions, we obtain our desired relation, given by

$$Kn_{x,w} = \sqrt{\frac{\pi\gamma}{2}} \frac{M_\infty}{Re_{x,\infty}} \left(\frac{p_\infty}{p_w}\right)\left(\frac{T_w}{T_\infty}\right)^{1/2+\omega} \tag{13}$$

It is now appropriate to review the pressure and temperature ratios appearing in Eq. (13). We will use the approximation for pressure in the weak interaction regime, i.e., where $p_w/p_\infty \approx p_e/p_\infty \approx 1 + a\bar{\chi}_\infty \approx 1$.[13] This approximation has also been used for heat transfer analyses in the strong interaction regime, stemming from observations that streamwise pressure gradients apparently do not affect the thermal boundary layer.[17] On this basis, we write

$$Kn_{x,w} = \sqrt{\frac{\pi\gamma}{2}} \frac{M_\infty}{Re_{x,\infty}} \left(\frac{T_w}{T_\infty}\right)^{1/2+\omega} \tag{14}$$

which was cited as Eq. (6) above.

For the temperature ratio appearing in Eq. (14), we note that the plate temperature is a fixed, independent quantity in experiments employing a cold, constant-temperature plate. It is usually presented experimentally as T_w/T_0, where T_0 is the stagnation temperature. From the known relation for T_0, we can write

$$\frac{T_w}{T_\infty} = \left(\frac{T_w}{T_0}\right)\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) \tag{15}$$

In experiments where it is appropriate to assume that the plate is insulated, a good approximation for the plate temperature, empirically demonstrated by many authors,[12,15] is the expression for the recovery temperature of an insulated plate in laminar, incompressible flow.[12] It is given by

$$(T_w/T_\infty) = 1 + [(\gamma - 1)/2] \sqrt{Pr_\infty} M_\infty^2 \tag{16}$$

In either case, from Eqs. (15) and (16), we see that T_w/T_∞ is proportional to M_∞^2 when the Mach number is large. The dependence of $Kn_{x,w}$ on the Mach number and the Reynolds number for large M_∞, therefore, varies as

$$Kn_{x,w} \propto \frac{M_\infty^{2\omega+1}}{Re_{x,\infty}} \tag{17}$$

Comparing the functional dependence of the three parameters defined by Eqs. (1) and (3), and the square root of Eq. (17), one finds that they have essentially the same Reynolds number dependence, but significantly differ in their Mach number dependencies. We note for Eq. (17) that an explicit dependence on the Mach number arises from the physics relating the local mean free-path to freestream variables, and an implicit dependence arises from the role of the wall temperature.

A difference in the formulation of $Kn_{x,w}$ from $\bar{\chi}_\infty$, and $\bar{V}_\infty$, lies in its connection of the freestream static temperature to the plate temperature, rather than the freestream stagnation temperature for high surface temperature conditions. We propose to use Eq. (15) or Eq. (16) for the wall temperature ratio in defining $Kn_{x,w}$, so that we may determine whether a broader correlation would be found for drag and heat transfer data than by simply assuming $Pr_\infty = 1$ and large Mach numbers.[13] When computing $\bar{\chi}_\infty$ or $\bar{V}_\infty$, one must decide where to evaluate the constant C in the boundary layer for the linear viscosity-temperature fit.[3,5,6,9] We have based the parameter $Kn_{x,w}$ on the plate temperature, regardless of the temperature variation normal to the surface within the boundary layer. Disregarding this normal boundary-layer temperature variation, is in keeping with the concept that the kinetic effect of gas-surface collisions principally determines the drag and heat transfer to the plate.

When freestream conditions become even more rarefied than those associated with the viscous interaction regimes, one must distinguish between the mean free-path length associated with collisions between surface-reflected molecules and the incident flow, and that associated with collisions between two newly-reflected molecules.[18] Our interpretation of λ_w is associated with the occurrence of this latter type of interaction. Certainly, such a distinction would be necessary in the nearly free molecule "first collision" regime, where some investigators reason that the former mean free-path length is the dominant length scale.[18-20] Arguably, the distinction could be important throughout the entire transitional "slip flow" regime, which usually borders on the strong interaction zone. Physically, in slip flow at the molecular level (as a molecule collides with a solid surface) diffuse scattering still dominates; that is to say, that there is no preferred direction associated with the re-emitted molecule.[21] But upon re-emission, there is little probability that it will collide with neighboring molecules in the vicinity of where it was emitted. Therefore, a sampling of molecules in this region reveals appreciable bulk velocity even as the solid surface is approached closely. The physical behavior of slip flow is controlled by the fact that the flow is locally no longer collision-dominated, in contrast to the continuum limit. This suggests that the correlating ability of $Kn_{x,w}$ diminishes when the flowfield becomes sufficiently rarefied, and the distinction between incident and reflected molecules becomes important.

A general theory for rarefied flows remains unknown, and specific expressions for the drag coefficient C_D and the Stanton number C_H as functions of M_∞ and $Re_{x,\infty}$ (or a suitable Knudsen number) are not available. Straightforward plots of experimental data in these basic variables yield families of curves. In an attempt to reduce this variation, one may replace the dependent variables with those occurring in the known theory for the free-molecule limit. Basing the drag coefficient on two-sided drag, the area of the plate, and by assuming fully-diffuse reflection, the appropriate expression in the free-molecule limit is given by[2]

$$C_{D,\mathrm{fm}} = \sqrt{(8/\pi\gamma)}(1/M_\infty) \tag{18}$$

Therefore, by using the product $C_D M_\infty$ as a dependent variable, the single value of $\sqrt{8/\pi\gamma}$ is obtained for all Mach numbers and plate temperatures as the free-molecule limit is approached.

Finding a corresponding limiting value for the heat transfer coefficient is more complicated. The free-molecule equation for the heat transfer coefficient, using the assumptions of Eq. (18), is given by[13]

$$C_{Q,\mathrm{fm}} \equiv \left(\frac{Q}{\frac{1}{2}\rho_\infty U_\infty^3 A}\right)_{\mathrm{fm}} = \frac{1}{M_\infty}\sqrt{\frac{2}{\pi\gamma}}$$
$$\cdot\left\{1 + \frac{\gamma+1}{\gamma(\gamma-1)}\left[\frac{1}{M_\infty^2}\left(\frac{T_w}{T_0}\right)\left(\frac{1}{M_\infty^2} + \frac{\gamma-1}{2}\right)\right]\right\} \tag{19}$$

The quantity Q is defined as the amount of energy delivered to the plate from the fluid per unit time. In the limit of large Mach numbers, we may write

$$\lim_{M_\infty\to\infty} (C_Q M_\infty)_{\mathrm{fm}} = \sqrt{\frac{2}{\pi\gamma}}\left[1 - \frac{\gamma+1}{2\gamma}\left(\frac{T_w}{T_0}\right)\right] \tag{20}$$

Finally, under "hyperthermal" conditions, which are distinguished by large Mach numbers and very cold surface temperatures,[18] we find the limiting expression

$$\lim_{\substack{M_\infty\to\infty \\ T_w\to 0}} (C_Q M_\infty)_{\mathrm{fm}} = \sqrt{(2/\pi\gamma)} \tag{21}$$

Only under hyperthermal, free-molecule conditions is the product $C_Q M_\infty$ a constant. Let us denote the general free-molecule product given by M_∞ and Eq. (19) as $(C_Q M_\infty)_{\mathrm{fm}}$. In order to correlate experimental heat transfer data so that a single value is reached in the free-molecule limit (assuming complete thermal accommodation) we simply take the experimental values of $C_Q M_\infty$ and multiply by a ratio made up of the hyperthermal free-molecule limit (Eq. (21)) over the free-molecule limit given by Eq. (19). The modified value of $C_Q M_\infty$ is introduced as the ordinate variable

$$(C_Q M_\infty)^* \equiv \left(\frac{\sqrt{2/\pi\gamma}}{(C_Q M_\infty)_{\mathrm{fm}}}\right) C_Q M_\infty \tag{22}$$

The parameter C_Q introduced here is related to the Stanton number C_H by

$$C_Q = \xi C_H \tag{23}$$

where

$$\xi \equiv \left(1 + \frac{2}{\gamma-1}\frac{1}{M_\infty^2}\right)\left(1 - \frac{T_w}{T_0}\right) \tag{24}$$

assuming a constant ratio of specific heats γ. Notice that in the hyperthermal limit, a "Reynolds Analogy" exists between the drag and heat transfer coefficients in Eqs. (18) and (21), where $C_Q = C_D/2$.

Results

The following figures illustrate the abilities of the two most commonly used hypersonic variables, $\bar{V}_\infty$ and $\bar{\chi}_\infty$, along with the proposed hypersonic parameter $Kn_{x,w}$, to correlate drag and heat transfer data resulting from many experimental investigations of supersonic, low-density flow of diatomic gases over flat plates at zero incidence with sharp leading edges.[1-6,9-11] Those experiments having surface temperatures near the theoretical recovery value, will be referred to as "hot wall" data, and those having much lower surface temperatures, shall be referred to as "cold wall" data. The values used for C in $\bar{\chi}_\infty$ and $\bar{V}_\infty$, when they were not given by the investigators, were evaluated at a T_{ref} estimated either by using Eq. (16) for insulated plates, or by using the averaging process suggested by Cheng for cold wall cases.[9] In an attempt to provide continuity between plots using different independent variables, and to measure their relative effectiveness, it was decided to present them with the same Reynolds number dependence in the numerator. The final forms of the variables plotted in the following figures are C_D and C_H vs $1/\bar{\chi}_\infty$ and $1/\bar{V}_\infty$, and $C_D M_\infty$ and $(C_Q M_\infty)^*$ vs $1/\sqrt{\overline{Kn}_{x,w}}\cdot Kn_{x,w}$ is replaced here by the reduced Knudsen number $\overline{Kn}_{x,w}$, where the latter differs from $Kn_{x,w}$ by dropping the numerical prefix $\sqrt{\pi\gamma/2}$. An added benefit from this presentation is that the data appear more like the familiar C_D vs $Re_{x,\infty}$ plots depicted in continuum theory. As noted earlier, we see that the Mach number dependence of $Kn_{x,w}$ lies between $\bar{\chi}_\infty$ and $\bar{V}_\infty$, although it is closer to $\bar{V}_\infty$.

Figure 1 displays hot wall drag coefficient data sets for Mach numbers ranging from 2.5 to 20 and Reynolds numbers of 34 to 2.4×10^6. Figure 1a shows that when the viscous interaction parameter $\bar{\chi}_\infty$ is used, the resulting correlation is very poor. (This trend persists for all data presented herein, as has been noted by others.[22,23]) When using the slip parameter $\bar{V}_\infty$, as

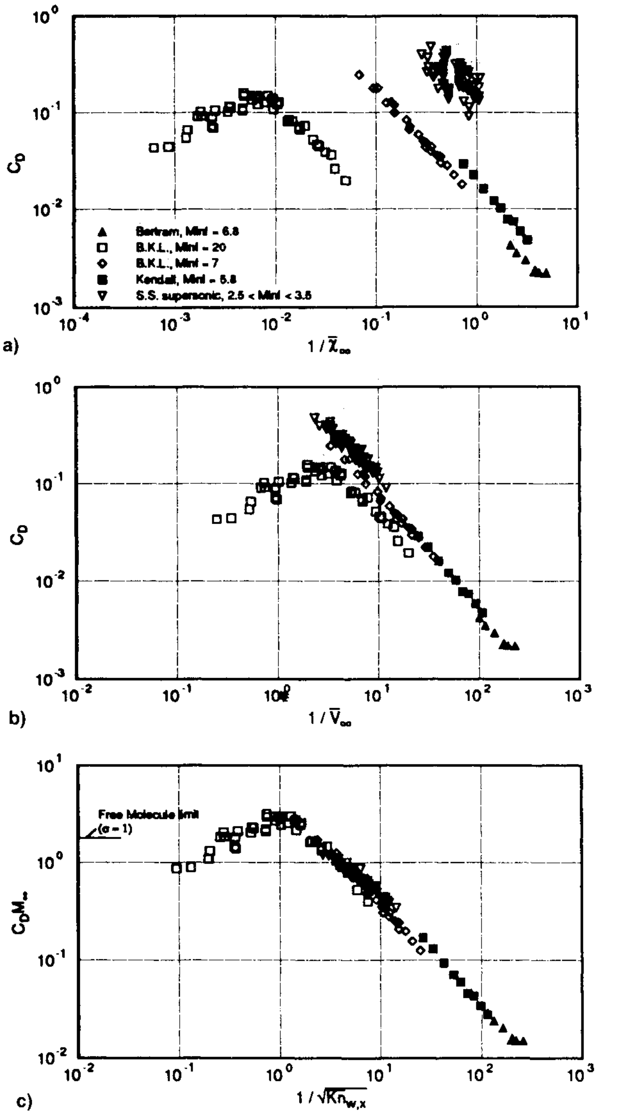

Fig. 1 Comparison of hot wall drag coefficient data vs three correlation parameters. Presented as C_D vs the reciprocal of a) the viscous interaction parameter $\bar{\chi}_\infty$, and b) the slip parameter $\bar{V}_\infty$, and $C_D M_\infty$ vs the reciprocal of (c) the square root of the reduced Knudsen number $\overline{Kn}_{x,w}$.

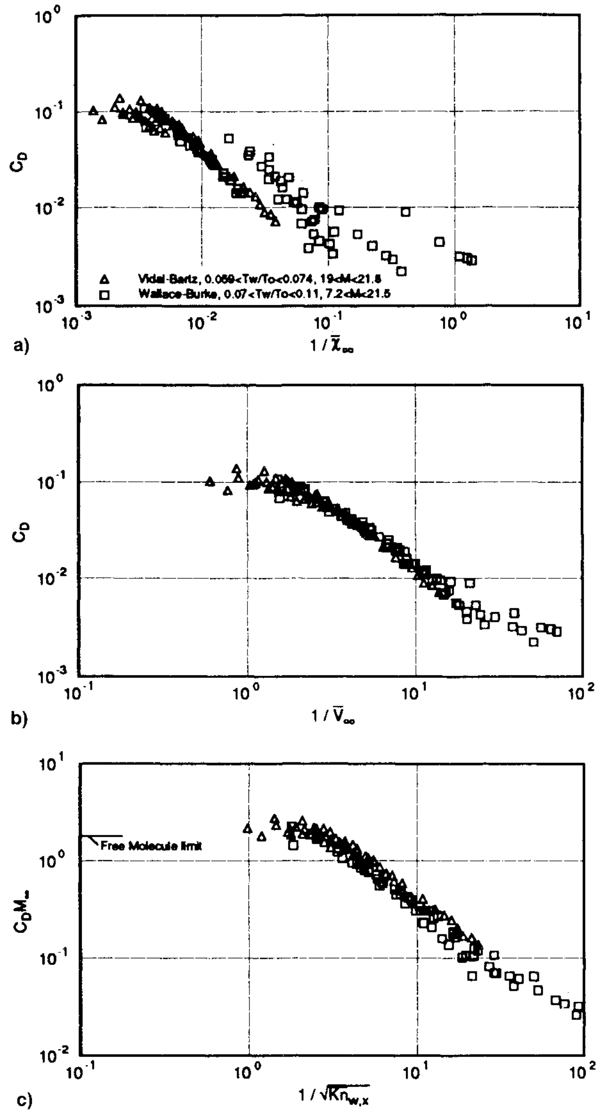

Fig. 2 Comparison of cold wall drag coefficient data vs three correlation parameters. Presented as C_D vs the reciprocal of a) the viscous interaction parameter $\bar{\chi}_\infty$, and b) the slip parameter $\bar{V}_\infty$, and $C_D M_\infty$ vs the reciprocal of c) the square root of the reduced Knudsen number $\overline{Kn}_{x,w}$.

seen in Figure 1b, there is good correlation only at moderate Mach numbers. A Mach number data dependency is evident here, since the drag coefficient drops at constant $\bar{V}_\infty$ with increasing M_∞. Figure 1c shows the superior features of $\overline{Kn}_{x,w}$ over the entire range of data. Only the trailing portion of Böttcher et al.[1] $M_\infty = 7$ data set lies outside of the locus, and the apparent evidence of incomplete momentum accommodation in the transitional regime for their $M_\infty = 20$ data is left unexplained by the authors.[1]

Figures 2a–c show a similar presentation for the cold wall data sets of Vidal and Bartz[5] and Wallace and Burke.[6] Mach numbers range from 7 to 22, Reynolds numbers from 20 to 2.2×10^5, and wall to stagnation temperature ratios from 6 to 11%. In Figure 2a, it is again observed that $\bar{\chi}_\infty$ is a poor choice, although its use here is more effective than for the hot wall data of Figure 1a. The slip parameter $\bar{V}_\infty$ (Figure 2b) apparently demonstrates very good correlation for values less than 0.1, which in terms of $\bar{\chi}_\infty$, places those values well in the strong interaction zone. The corresponding plot of this data vs $\overline{Kn}_{x,w}$ (Fig. 2c) shows that it is not well-correlated for $\overline{Kn}_{x,w} < 0.01$. Above this value, $\overline{Kn}_{x,w}$ is almost as effective as $\bar{V}_\infty$ appears to be.

A closer examination of Fig. 2b, however, reveals that the slope associated with the Vidal and Bartz data plotted against $\bar{V}_\infty$, appears steeper than that associated with Wallace and Burke's. This is because the Wallace and Burke data plotted here consists of a few data points taken at each set of freestream conditions, where Mach numbers ranged from 7 to 22. For Vidal and Bartz's data, Mach numbers only varied from 19 to 22. Plotting Wallace and Burke's data with different symbols for each Mach number, a Mach number dependency becomes evident. The values of C_D at $M_\infty = 7$ for a given $\bar{V}_\infty$ are 50–100% greater than those at $M_\infty = 11.6$ and 15.5. This Mach number dependency is eliminated in Fig. 2c when using $\overline{Kn}_{x,w}$. The presentation of additional cold wall drag data sets with greater freestream Mach number and stagnation temperature ratio variation would prove enlightening.

Figure 3 shows results from various cold wall heat transfer investigations. Use of $\bar{\chi}_\infty$ in Fig. 3a results in the same poor level of correlation obtained for the data of Fig. 2a. Using $\bar{V}_\infty$ (Fig. 3b) results in better correlation, but again there is a data dependency on the Mach number and stagnation temperature ratio, which is obscured by the Wallace and Burke data. The Stanton number tends to increase at constant $\bar{V}_\infty$ with increasing M_∞ and T_w/T_0. This problem appears to be overcome in Fig. 3c by plotting the heat transfer data against $\overline{Kn}_{x,w}$, which exhibits fairly good correlation throughout the entire viscous interaction range.

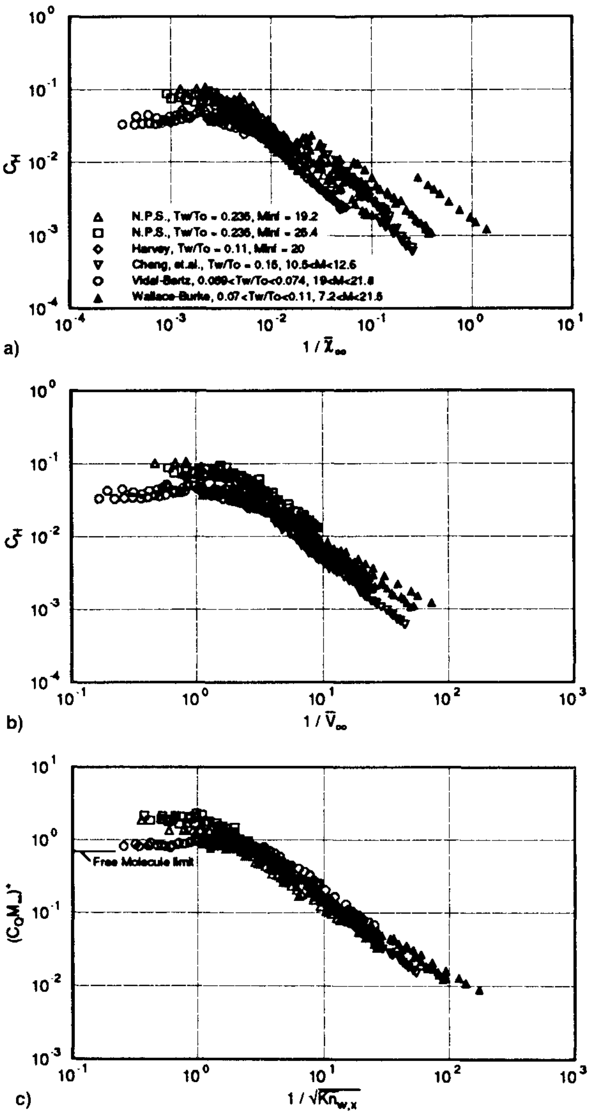

Fig. 3 Comparison of cold wall heat transfer coefficient data vs three correlation parameters. Presented as C_H vs the reciprocal of a) the viscous interaction $\bar{\chi}_\infty$, and b) the slip parameter $\bar{V}_\infty$, and $(C_Q M_\infty)^*$ vs the reciprocal of c) the square root of the reduced Knudsen number $\overline{Kn}_{x,w}$.

In the results presented above, it was considered desirable to use $\sqrt{\overline{Kn}_{x,w}}$ as a correlating parameter in order to make comparisons with the viscous interaction parameter $\bar{\chi}_\infty$ and the slip parameter $\bar{V}_\infty$. Generally, it would be more straightforward to plot results using $Kn_{x,w}$ as presented in Eq. (14).

Conclusions

The independent and dependent variables associated with drag and heat transfer to a flat plate at zero incidence in high-speed, rarefied flow were analyzed anew to reflect the importance of kinetic effects occurring near the plate surface, rather than following arguments normally used to describe continuum, higher density flowfields. A new parameter, the wall Knudsen number $Kn_{x,w}$, was introduced and used to correlate published drag and heat transfer data for a wide range of experimental conditions. The new parameter was shown to provide better correlation than either the viscous interaction parameter $\bar{\chi}_\infty$ or the widely-used slip parameter $\bar{V}_\infty$.

Acknowledgments

This work was supported by the Air Force Office of Scientific Research under Grant AFOSR 90-0232, and in part by NASA Grant NCA2-495.

References

[1]Böttcher, R. D., Koppenwallner, G., and Legge, H., "Flat Plate Skin Friction in the Range Between Hypersonic Continuum and Free Molecular Flow," *Rarefied Gas Dynamics*, edited by J. L. Potter, Vol. 51, Pt. I, Progress in Astronautics and Aeronautics, DFVLR Press, Porz-Wahn, Germany, 1977, pp. 349–359.

[2]Schaaf, S. A., and Sherman, F. S., "Skin Friction in Slip Flow," *Journal of the Aeronautical Sciences*, Vol. 21, No. 2, 1954, pp. 85–90, 144.

[3]Kendall, J. M., "An Experimental Investigation of Leading-Edge Shock-Wave—Boundary-Layer Interaction at Mach 5.8," *Journal of Aeronautical Sciences*, Vol. 24, No. 1, 1957, pp. 47–56.

[4]Bertram, M. H., "Boundary-Layer Displacement Effects in Air at Mach Numbers of 6.8 and 9.6," NASA TR R-22, 1959.

[5]Vidal, R. J., and Bartz, J. A., "Surface Measurements on Sharp Flat Plates and Wedges in Low-Density Hypersonic Flow," *AIAA Journal*, Vol. 7, No. 6, 1969, pp. 1099–1109.

[6]Wallace, J. E., and Burke, A. F., "An Experimental Study of Surface and Flow Field Effects in Hypersonic Low Density Flow over a Flat Plate," *Rarefied Gas Dynamics*, Supplement 3, edited by J. H. de Leeuw, Vol. 1, Academic Press, New York, 1965, pp. 487–507.

[7]Kemp, J. H., Jr., "Hypersonic Viscous Interaction on Sharp and Blunt Inclined Plates," *AIAA Journal*, Vol. 7, No. 7, 1969, pp. 1280–1289.

[8]Henderson, A., Jr., Watson, R. D., and Wagner, R. D., Jr., "Fluid Dynamic Studies to $M = 41$ in Helium," *AIAA Journal*, Vol. 4, No. 12, 1966, pp. 2117–2124.

[9]Cheng, H. K., Hall, J. G., Golian, T. C., and Hertzberg, A., "Boundary-Layer Displacement and Leading-Edge Bluntness Effects in High-Temperature Hypersonic Flow," *Journal of Aerospace Sciences*, Vol. 28, No. 5, 1961, pp. 353–381, 410.

[10]Harvey, W. D., "Effects of Leading-Edge Bluntness on Pressure and Heat-Transfer Measurements over a Flat Plate at a Mach Number of 20," NASA TN D-2846, Oct. 1965.

[11]Nagamatsu, H. T., Pettit, W. T., and Sheer, R. E., Jr., "Heat Transfer on a Flat Plate in Continuum to Rarefied Hypersonic Flows at Mach Numbers of 19.2 and 25.4," NASA CR-1692, Nov. 1970.

[12]Schlichting, H., *Boundary-Layer Theory*, 7th ed., McGraw-Hill, New York, 1979, Chaps. 12–13.

[13]Probstein, R. F., and Hayes, W. D., *Hypersonic Flow Theory*, Academic Press, New York, 1959, Chaps. 9–10.

[14]Talbot, L., "Criterion for Slip near the Leading Edge of a Flat Plate in Hypersonic Flow," *AIAA Journal*, Vol. 1, No. 5, 1963, pp. 1169–1171.

[15]Anderson, J. D., Jr., *Hypersonic and High Temperature Gas Dynamics*, McGraw-Hill, San Franciso, CA, 1989, Chaps. 6–7.

[16]Kogan, M. N., *Rarefied Gas Dynamics*, Plenum Press, New York, 1969, Chaps. 4–5.

[17]Cox, R. N., and Crabtree, L. F., *Elements of Hypersonic Aerodynamics*, Academic Press, New York, 1965, Chaps. 7–8.

[18]Probstein, R. F., "Shock Wave and Flow Field Development in Hypersonic Re-Entry," *American Rocket Society Journal*, Vol. 31, Feb. 1961, pp. 185–194.

[19]Baker, R. M. L., Jr., and Charwat, A. F., "Transitional Correction to the Drag of a Sphere in Free Molecule Flow," *Physics of Fluids*, Vol. 1, No. 2, 1958, pp. 73–81.

[20]Kinslow, M., and Potter, J. L., "Drag of Spheres in Rarefied Hypervelocity Flow," *AIAA Journal*, Vol. 1, No. 11, pp. 2467–2473.

[21]Kennard, E. H., *Kinetic Theory of Gases*, McGraw-Hill, New York, 1938, Chap. 8.

[22]Wilhite, A. W., Arrington, J. P., and McCandless, R. S., "Performance Aerodynamics of Aero-Assisted Orbital Transfer Vehicles," AIAA Paper 84-0406, Reno, NV, Jan. 1984.

[23]Woods, W. C., Arrington, J. P., and Hamilton, H. H., II, "A Review of Preflight Estimates of Real-Gas Effects on Space Shuttle Aerodynamic Characteristics," NASA CP-2283, 1983, pp. 309–346.